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MARKOV PROCESSES APPLIED TO CONTROL, REPLACEMENT, AND SIGNAL AN--ETC(U)
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Final Research Report

on

MARKOV PROCESSES
APPLIED TO CONTROL, REPLACEMENT, AND SIGNAL ANALYSIS

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by

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B. DESCRIPTION OF WORK COMPLETED

[1] Hunt Processes. This is the final link in the major work on the characterization of Markov processes. In [8], we had characterized all semimartingale Hunt processes taking values in R^n in terms of Wiener processes and Poisson random measures. Thrust of the present work is to reduce an arbitrary Hunt process X taking values in an arbitrary space E to the case studied before. The main idea is to construct a function $f: E \to \hat{E} \subset [0,1]^\infty$ such that i) f is one-to-one and bimeasurable, ii) $f(X)$ is again a Hunt process, and iii) each component of $f(X)$ is a semimartingale. Such a construction is carried out by using the martingale additive functionals generated by a countable collection of smooth functions that serve as an orthonormal basis for the set of all martingales. In addition to yielding the desired representation, the method gives representations of all martingales and semimartingales defined on the probability base of the given Markov process.

[2] Queues with random intensities. Although written as an answer to a problem in queueing theory, the major idea of this paper is in fact the construction of point processes. The problem is to construct a point process whose dual predictable projection is specified somehow. The heart of the paper is the observation that all such problems can be transformed into a problem concerning the solution of certain stochastic integral equations driven by Poisson random measures. This method is purely sample-path-theoretic and replaces the need to use Girsanov's theorem on changing probability measures. Thus, an implicit analytic problem is solved by an explicit construction. Further, the explicit construction seems well-suited for use in simulation studies.

[3] Martingales, Brownian motion, stochastic integrals. This is an introduction to the theory of stochastic integrals using the simple cases of discrete-time and Poisson integrals as starting points. In particular, many of the fairly sophisticated results concerning Brownian motions are made simple by approximating the Brownian motion by using Poisson random measures. Also, we study the construction of stochastic integrals by the original method used by Ito, and show that it is capable of handling most situations with certain appropriate modifications. These notes are expository in nature, and will be used in a forthcoming book.

[4] Stochastic means. Let X be a Brownian motion in a Riemannian manifold and f be a smooth function. The expectation of $f(X_{T_\varepsilon})$ for the exit time T_ε from a geodesic ball of radius ε, suplies information concerning the structure of the manifold around the ball. This paper obtains the first three terms of the asymptotic expansion of that expectation, and shows that it coincides with the usual mean value only if the manifold is Einsteinian.

[5] Inverse problems. Let M be a surface inbedded in $\mathbf{R}^3$ so that every geodesic disc of radius ε has area $\pi \varepsilon^2$. Then, M is flat in the intrinsic sense that every small disc can be mapped isometrically to a flat disc in the plane $\mathbf{R}^2$. Generalizing this result to higher dimensional manifolds proved difficult to geometers. By replacing the concept of "area" by measures having to do with Brownian motion, one gets around some of the difficulties. First and second moments of the exit time (from balls of radius ε) of the Brownian motion give effective characterizations of Euclidean spaces and symmetric spaces with rank one.

[6] Brownian motion and Riemannian geometry. This is an expository paper on the interactions between Brownian motion and analysis on manifolds. In many geometric problems, the heat flow problems and its associated generator, the Laplacian, started to play an important role during recent research. Since heat flow is the macroscopic phenomenon corresponding to the microscopic process of Brownian motion, it is natural to investigate the interactions directly.

[7] Mean exit time from geodesics. This is a continuation of [4] and [5]. The mean exit time for a Brownian motion from a small geodesic ball is studied, an asymptotic expansion is given, and an explicit formula is provided when the manifold is symmetric and has rank one. The coefficients in the power series expansion are all expressible in terms of the invariants of the curvature operator, and thus, provide valuable information on the structure of the manifold around the geodesic.

[8], [9], [10] were described in the previous report, for the year 1980-81.

[11] Seminar on Stochastic Processes 1982. This volume will consist of about half the papers presented during a three-day seminar held at Northwestern University in March 1982. This was the second such seminar. The aim of these seminars is to bring together a small group of distinguished researchers and to provide an informal atmosphere for them to discuss their current work. The invited participants in this year's seminar were:

B. ATKINSON - Los Angeles

R. BASS - Seattle

K. BICHTELER - Austin

D. BURKHOLDER - Urbana

K.L. CHUNG - Palo Alto

J.L. DOOB - Urbana

C. DOLEANS-DADE - Urbana

H. FÖLLMER - Zürich

R.K. GETOOR - San Diego

J. GLOVER - Rochester

J. MITRO - Cincinnati

D. MONRAD - Urbana

E. PERKINS - Vancouver

J. PITMAN - Berkeley

Z. POP-STOJANOVICH - Gainesville

M.J. SHARPE - San Diego

T.C. SHIH - Ann Arbor

J. WALSH - Vancouver

The following is the list of contributors (and the titles of their work) to the volume being edited for publication:

B.W. ATKINSON. Germ fields and a converse to the strong Markov property.

B.W. ATKINSON and J.B. MITRO. Applications of Revuz and Palm type measures for additive functionals in weak duality.

R.F. BASS..Occupation times of d-dimensional semimartingales.

K. BICHTELER and D. FONKEN. A simple version of the Malliavin calculus in dimension N.

K.L. CHUNG. An inequality for boundary value problems.

R.K. GETOOR. Excursions and forward times.

J. GLOVER. Identifying Markov processes up to time change.

J. GLOVER. Topics in energy and potential theory.

H. KASPI. Regenerative systems and Markov additive processes.

D. MONRAD. On the p-variations of Gaussian random fields with separable increments.

Z. POP-STOJANOVICH and K.M. RAO. Remarks on energy.

J.B. WALSH. Stochastic integration with respect to local time.

A. SUMMARY

The following is the list of work completed during June 1981 - May 1982 under this grant.

[1] E. ÇINLAR and J. JACOD. Hunt processes satisfy stochastic integral equations driven by Wiener processes and Poisson random measures after a time change. To appear.

[2] _____. Queues with arrivals and sevices having random intensities. To appear.

[3] _____. Martingales, Brownian motion, and stochastic integrals.

[4] M. PINSKY. Moyennes stochastique sur une variété riemannienne. C.R. Acad. Sci. de Paris, t. 292, 991-994.

[5] _____. Inverse problems in stochastic Riemannian geometry. Proceedings of Second Bad Honnef Workshop on Stochastic Differential Systems. Springer-Verlag, Lecture Notes in Mathematics. To appear.

[6] _____. Brownian motion and Riemannian geometry. Proc. Special Year in Differential Geometry, Univ. of Maryland. Birkhäuser, Boston, 1982. In press.

[7] _____ and A. GRAY. The mean exit time from a small geodesic ball in a Riemannian manifold. To appear.

Copies of these papers are attached. In addition, copies are attached of the following published work reported to AFOSR earlier.

[8] E. ÇINLAR and J. JACOD. Representation of semimartingale Markov processes in terms of Wiener processes and Poisson random measures. In Seminar on Stochastic Processes 1981, pp. 159-242. Birkhäuser, Boston, 1981.

[9] _____ and _____. Semimartingales defined on Markov processes.

[10] _____, K.L. CHUNG and R.K. GETOOR, eds. Seminar on Stochastic Processes 1981. Birkhäuser, Boston, 1981.

Finally, we report on the Seminar on Stochastic Processes 1982 held in Evanston during March 1982. The volume to appear is under preparation:

[11] E. ÇINLAR, K.L. CHUNG, and R.K. GETOOR, eds. Seminar on Stochastic Processes 1982. Birkhäuser, Boston, 1982. To appear.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes the work completed under the grant in the following areas: (1) Hunt processes; (2) queues with random intensities; (3) martingales, Brownian motion, stochastic integrals; (4) stochastic means; (5) inverse problems; (6) Brownian motion and Riemannian geometry; (7) mean exit time from geodesics; and (8) seminar on stochastic processes 1982. Also included are a list of the participants in the seminar; a list of contributors and the titles of their work to the proceedings of the seminar; and a list of work completed during the last year of the grant.

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